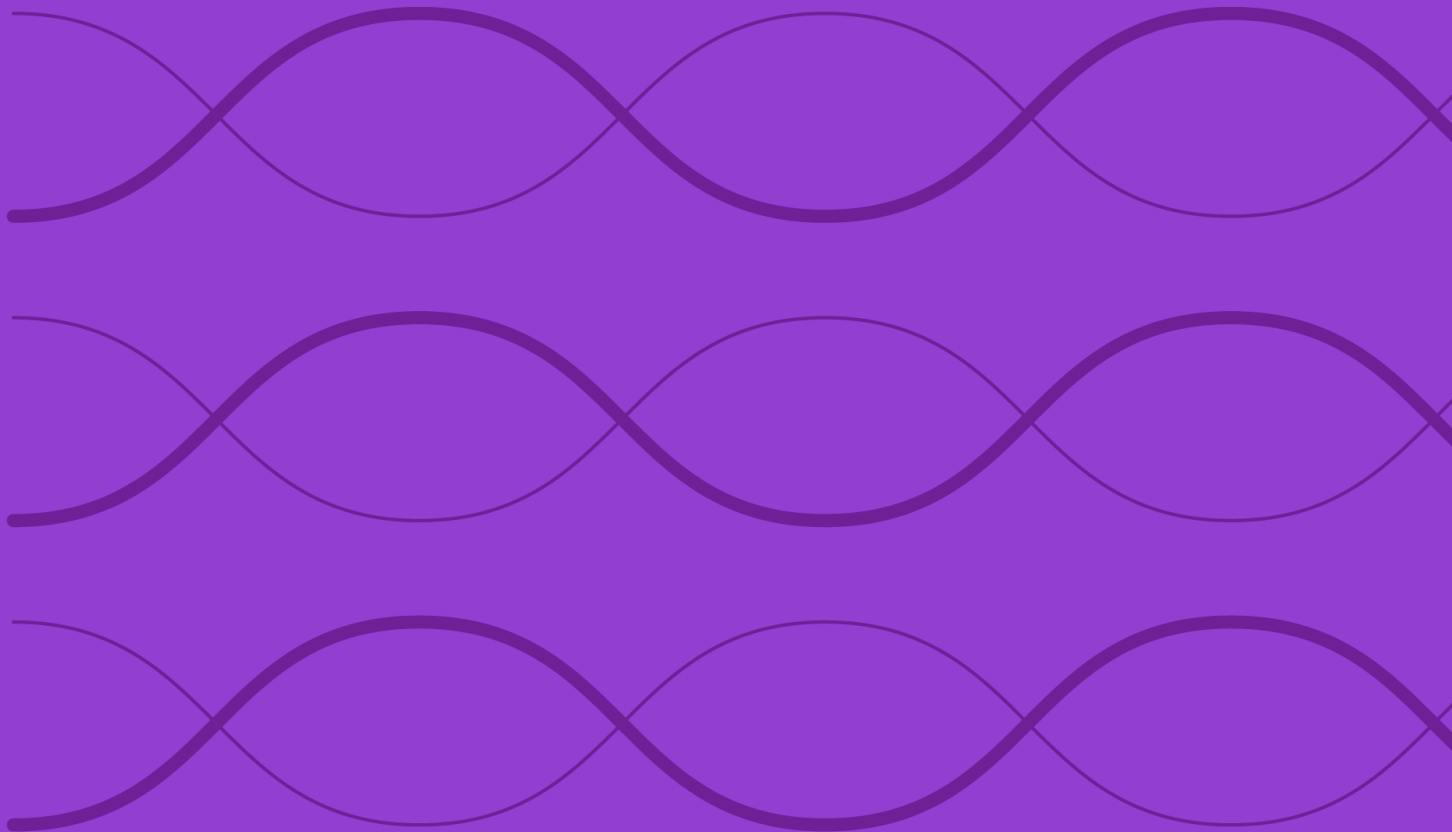


Construction Rules for the Morningstar Indexes Distance to Default



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Black-Scholes-Merton structural model governing equations

We may use equity prices and fundamentals to estimate the default probability of a company. It has been shown (Merton 1974) that the company's equity may be modelled as an option on the assets of the company. We may then use the solution to the Black-Scholes-Merton equation for a European call option (1-4) to value the equity. This yields a structural model that relates the equity value to the value of the unobservable asset values.

$$(1) E_0 = D_0 + A_0 \Phi(d_1) - L_T e^{-r_f T} \Phi(d_2)$$

Where

$$(2) \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-z^2/2} dz$$

And

$$(3) d_1 = \frac{\ln\left(\frac{A_0 - D_0}{L_T}\right) + \left(r_f + \frac{\sigma_A^2}{2}\right)T}{\sigma_A \sqrt{T}}$$

$$(4) d_2 = d_1 - \sigma_A \sqrt{T} = \frac{\ln\left(\frac{A_0 - D_0}{L_T}\right) + \left(r_f - \frac{\sigma_A^2}{2}\right)T}{\sigma_A \sqrt{T}}$$

The distance to default is d_2 and the risk-neutral probability that the company will default on the debt is $\Phi(-d_2)$. For our purposes we will assume a constant time horizon of one year, setting $T=1$.

Data point definitions

Data point definitions used as inputs are all in local currencies.

E_0 = the current value of the aggregated company equity

D_0 = total dividends to be paid within one year using forward dividend, else historical dividends

L_T = liabilities due within one year using short term liabilities, else long term liabilities

r_f = logarithmic risk-free rate (annual rate)

Data points calculated or solved for

σ_E = standard deviation of logarithmic equity returns (annual) (instantaneous value). This is accomplished by using the exponentially weighted moving average (EWMA) estimate.

A_0 = the current value of the firm's assets in local currency

σ_A = standard deviation of logarithmic asset returns (annual) (assumed to be constant)

Exponentially weighted moving average estimates of equity volatility

The equity volatility parameter, σ_E , is the most important parameter in the Merton model. Therefore, it is important to have an estimation methodology that responds quickly to recently observed changes in the volatility of equity returns. Exponentially weighted averaging is a standard method of achieving this.

Let P_t denote the closing share price on day t and r_t denote the daily logarithmic return on equity. We have:

$$(5) \quad r_t = \ln(P_t) - \ln(P_{t-1})$$

Let λ denote the exponential decay rate. We set λ to value in the range 0.97. We calculate the exponentially weighted average daily logarithmic return recursively as follows:

$$(6) \quad \mu_{1t} = (1 - \lambda)r_t + \lambda\mu_{1t-1}$$

We initialize $\mu_{10}=0$.

Similarly, we calculate the exponentially weighted average daily logarithmic return squared recursively as follows:

$$(7) \quad \mu_{2t} = (1 - \lambda)r_t^2 + \lambda\mu_{2t-1}$$

We initialize μ_{20} to an initial estimate of σ_E^2/NTY , NTY being the assumed number of trading days per year which we take to be 252.

Our estimate of the annualized standard deviation of logarithmic equity returns described below on day t is:

$$(8) \quad \hat{\sigma}_{Et} = \sqrt{NTY(\mu_{2t} - \mu_{1t}^2)}$$

Asset value and standard deviation of log asset returns

We now would like to solve for the asset value and asset volatility. In order to do this we need another equation to solve for these two variables. Luckily we may introduce another equation via Ito's Lemma in order to solve the set of nonlinear equations which implicitly define the asset value and volatility.

$$(9) \quad \sigma_E E_0 = \frac{\partial E_0}{\partial A_0} A_0 \sigma_A = \Phi(d_1) A_0 \sigma_A$$

Equations (1-4, 9) must now be solved. This may be done by recasting the problem as an optimization problem.

Let us write the zero set of equations (1) and (9) as

$$(10) \quad G(A_0, \sigma_A) = E_0 - D_0 - A_0 \Phi(d_1) + L_T e^{-r_f T} \Phi(d_2)$$

$$(11) \quad H(A_0, \sigma_A) = \sigma_E E_0 - \Phi(d_1) A_0 \sigma_A$$

This set of equations can be solved for by using a constrained optimization procedure (we use Nelder-Mead) using the following objective function:

$$(12) \quad \min_{A_0, \sigma_A} F(A_0, \sigma_A) = G^2 + H^2$$

Distance to Default and Probability of Default Calculation

Once we have solved for the previous values we can calculate the distance to default:

$$(13) \quad d_2 = d_1 - \sigma_A \sqrt{T} = \frac{\ln\left(\frac{A_0 - D_0}{L_T}\right) + \left(r_f - \frac{\sigma_A^2}{2}\right)T}{\sigma_A \sqrt{T}}$$

And the probability of default is:

$$(14) \quad \Phi(-d_2)$$

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