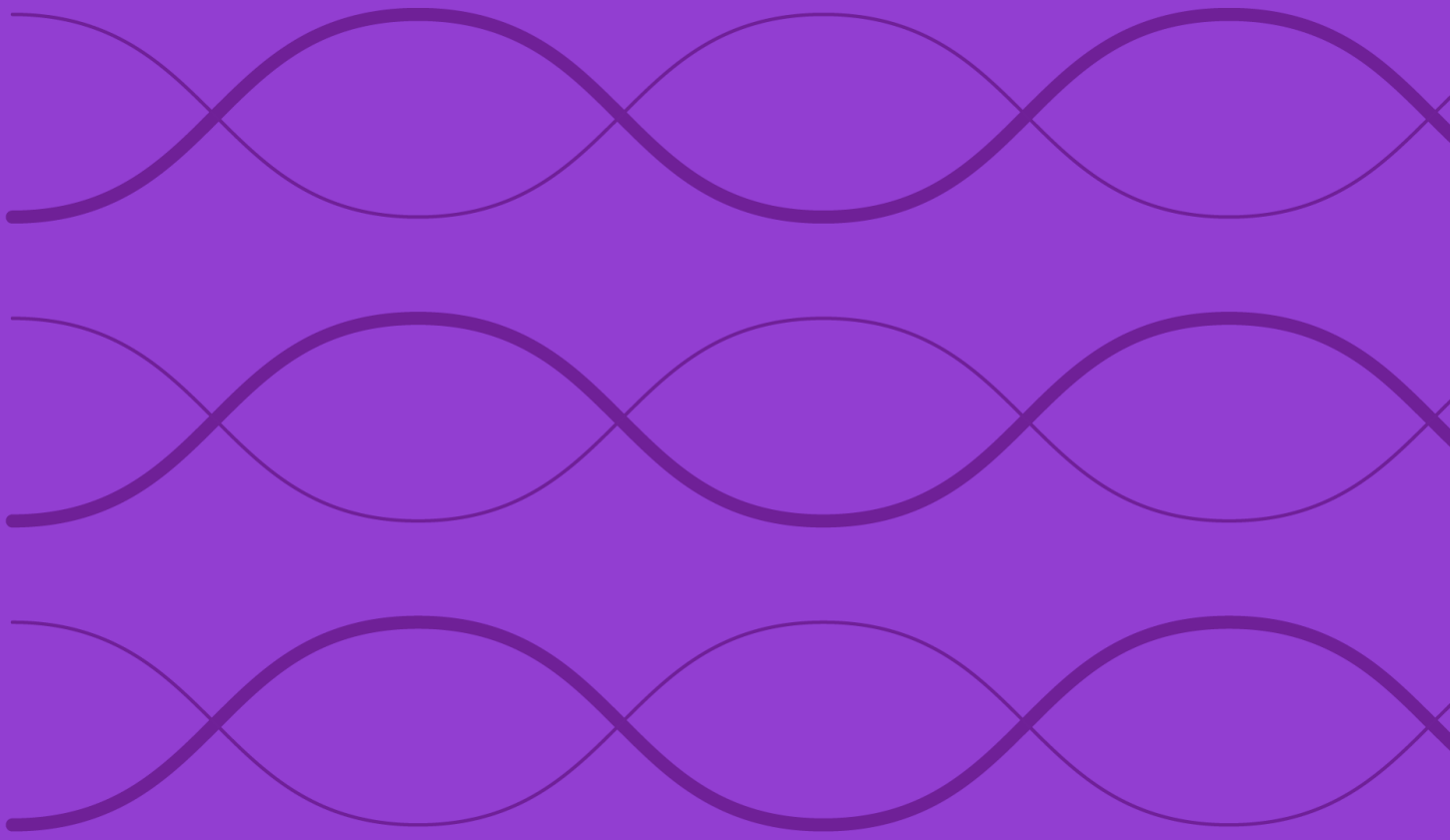
MORNINGSTAR Indexes

# Construction Rules for the Morningstar® Factor Tilt Indexes℠



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MORNINGSTAR®

## Overview

The Morningstar® Factor Tilt Indexes measure the performance of global equity markets with increased exposure toward small-capitalization and value-oriented stocks. These stocks have an overweighting in the indexes compared with their weight in their corresponding parent benchmarks, which are free-float, market-capitalization-weighted indexes. Likewise, large-cap or growth stocks are underweighted relative to their weights in the standard market-capitalization-weighted parent benchmarks. The indexes seek to capture both a value and size premium to achieve what the efficient-market camp refers to as systematic exposure to undiversifiable risk.

These indexes do not incorporate Environmental, Social, or Governance (ESG) criteria.

### Index Inception and Performance Start Date

The inception date and performance start date, when the first back-tested index values were calculated, are listed in Appendix 9.

## Index Construction

### Exhibit 1: Methodology Summary


### Starting Universe

At each reconstitution, securities for Morningstar US Market Factor Tilt Index are derived from the Morningstar US Market Extended Index, covering the top 99.5% of investable market capitalization. Similarly, at each reconstitution, securities for the Morningstar Emerging Markets and Morningstar Developed Markets ex-US Factor Tilt indexes are derived from the Morningstar Global Markets ex-US Index.

### Assigning Value Scores and Stock Style

- To qualify for inclusion, each stock must have a valid style score
- A stock's value orientation reflects the price investors are willing to pay for a share of some combination of the stock's prospective earnings, dividends, sales, cash flow, and book value.
- Value orientation is determined using the following three steps:
    - Calculate five prospective yields (earnings, dividend, cash flow, revenue, and book value) for each stock within each of the cap indexes.
    - Compute a cumulative value score for each security.
    - The total free-float market capitalization of a given size segment is divided equally among the three style segments.
- For complete rules on assigning value and style scores, refer to Appendix 3-6.

[1] See appendix 8 for parent benchmarks of each index in the family.

## Portfolio Construction

*Applying Factor Tilt*

- The purpose of these indexes is to allow investors to tilt their equity exposure toward a particular investment style while maintaining a low-turnover core portfolio.
- To arrive at the small-value tilt, the market is divided into four asymmetric quadrants. The value and size tilts are applied across the four quadrants, specifically overweighting the small-value quadrant relative to the other quadrants. With this approach, relative market weighting is maintained within each quadrant, resulting in a low-turnover core portfolio that captures long-term small-value premiums.
- For more details on tilt factor calculations, refer to Appendix 8.

*Number of Stocks*

The number of stocks in the index is variable, subject to the selection and eligibility criteria at the time of reconstitution.

*Index Weighting*

The indexes are tilt-weighted from float market capitalization weight based on the factor tilts described above. For more details, refer to Appendix 2 and the Morningstar Indexes Calculation Methodology rulebook.

## Index Maintenance and Calculation

### Scheduled Maintenance

The indexes are reconstituted, where, the membership is reset, semiannually. Adjustments are made on the Monday following the third Friday of June and December. If the Monday is a holiday, reconstitution occurs on the Tuesday immediately following. The market data used for reconstitution is as of the last trading day of April and October.

The indexes are rebalanced quarterly and implemented after the close of business on the third Friday of March, June, September, and December and the rebalance is effective the following Monday. If Monday is a holiday, the rebalance is effective on the following business day. The market data used for rebalancing is as of the last trading day of February, May, August, and November.

Refer to Appendix 2 for details on reconstitution and rebalancing.

Index files for the Morningstar US Market Factor Tilt Index are published according to the U.S. calendar schedule, while files for the Morningstar Emerging Markets Factor Tilt Index and Morningstar Developed Markets ex-US Index are published on the global calendar schedule. For more information, please refer to the Morningstar Indexes Holiday Calendar.

### Corporate Actions

The treatment of corporate actions will be as per the float market capitalization weighted indexes.
For more details, please refer to the "Treatment for float market capitalization indexes" section in the Morningstar Indexes Corporate Actions Methodology rulebook.

### Index Calculation and Price Data

Details about index calculations and price data can be found in their respective rulebooks: Morningstar Indexes Calculation Methodology and Equity Closing Prices Used for Index Calculation.

## Methodology Review and Index Cessation Policy

The index methodology is continually reviewed to ensure it achieves all stated objectives. These reviews take into account corporate action treatment, selection, and maintenance procedures. Subscribers to the index will be notified before any methodology changes are made. For more details, refer to the Morningstar Index Methodology Change Policy.

Morningstar also notifies all subscribers and stakeholders of the index that circumstances might arise that require a material change to the index, or a possible cessation of the index. Circumstances that could lead to an index cessation include, but are not limited to, market structure change, product definition change, inadequate supply of data, insufficient revenue associated with the index, insufficient number of clients using the index, and/or other external factors beyond the control of the Morningstar Index Committee.

Because the cessation of the index or benchmark index could disrupt subscriber products that reference this index, all subscribers are encouraged to have robust fallback procedures if an index is terminated. For more details, refer to the Morningstar Index Cessation Process.

## Data Correction and Precision

### Intraday Index Data Corrections

Commercially reasonable efforts are made to ensure the accuracy of data used in real-time index calculations. If incorrect price or corporate action data affects index daily highs or lows, it is corrected retroactively as soon as is feasible.

### Index-Related Data and Divisor Corrections

Incorrect pricing and corporate action data for individual issues in the database will be corrected upon detection. In addition, an incorrect divisor of an index, if discovered within two days of its occurrence, will always be fixed retroactively on the day it is discovered to prevent an error from being carried forward. Commercially reasonable efforts are made to correct an older error subject to its significance and feasibility.

For more details, refer to the Recalculation Guidelines.

### Computational and Reporting Precision

For reporting purposes, index values are rounded to two decimal places and divisors are rounded to appropriate decimal places.

## Exceptions

While Morningstar will seek to apply the method described above, the market environment, supervisory, legal, financial, or tax reasons may require an alternative approach to be adopted. A decision to take an alternative approach will be made by the relevant Morningstar Indexes Methodology Committee, and in all instances, the application of a nonstandard process will be reported to the Morningstar Indexes Oversight Committee.

# Appendixes

## Appendix 1: Modifications to the Rulebook

| Section | Description of Change | Update Date |
|---|---|---|
| Entire rulebook | • Moved to new template | August 2023 |

## Appendix 2: Glossary

| Terms | Description |
|---|---|
| **Reconstitution** | Each reconstitution involves the following:<br>• Updating the global market investable equity universe.<br>• Reviewing the economic segment- and country-level size segment breakpoints.<br>• Assigning companies to capitalization bands taking into account the buffer zones.<br>• Changes in index shares (free float, total shares outstanding, index-specific adjustment factor) of each constituent. |
| **Rebalance** | During each rebalancing, the following activities are undertaken:<br>• Changes in index shares (free float, total shares outstanding, index-specific adjustment factor) of each constituent.<br>• Addition of U.S. spin-offs/IPOs to the global markets index. |
| **Free Float** | The free float is defined as a security's outstanding shares adjusted by block ownership to reflect only truly tradable and investable shares. A security's outstanding shares are adjusted if an entity owns 5% or more of the security through one of the following types of block ownership:<br>• Cross-ownership—shares that are owned by other companies (including banks and life insurance companies).<br>• Government ownership—shares that are owned by governments (central or municipal) or their agencies.<br>• Private ownership—shares that are owned by individuals or families.<br>• Restricted shares—shares that cannot be traded during a certain time period.<br><br>However, a security's outstanding shares are not adjusted for institutional investors' holdings, which include, but are not limited to, the following categories:<br>• Custodian nominees<br>• Trustee companies<br>• Mutual funds<br>• Investment companies<br>• Pension fund holdings |

## Appendix 3: Determining the Value-Core-Growth Assignment for Common Stocks

*Basic Concepts*

A stock's value orientation and growth orientation are separate measures, each of interest to investors. As such, they are estimated using related but separate variables. Once estimated, depending on the purpose, they may be used individually or combined into a single value-core-growth, or VCG, orientation measure. For instance, style-based index construction requires the use of a single VCG measure for each stock.

VCG orientation is calculated within capitalization classes, or cap bands. That is, a stock is assigned to a cap band before its VCG orientation is determined. Two stocks that have similar financial ratios and growth prospects but are in different cap bands may be given different VCG assignments.

A high value orientation score (as defined below) indicates that a stock's price is relatively low, given the anticipated per share earnings, book value, revenue, and so forth that the stock provides to investors. A high price relative to these measures indicates that a stock's value orientation is weak but does not necessarily mean that the stock is growth-oriented.

A high growth orientation score (as defined below) indicates that a stock's per share earnings, book value, revenue, and so on are expected to grow faster than those of other stocks in the same cap band. A weak growth orientation does not necessarily mean that a stock has a strong value orientation.

It follows that an individual stock may have any combination of strong or weak growth and value characteristics. Where one set of characteristics is dominant, the stock can be classified accordingly. Where the stock's growth and value characteristics are similar in strength, the stock will be assigned a blend VCG orientation.

*Terminology and Notation*

The following short forms relate to the 10 factors used to determine a stock's VCG score:

| | | |
|---|---|---|
| e1/p | = | prospective earnings yield (forecast earnings per share for the current fiscal year, divided by current price per share) |
| r1/p | = | prospective revenue yield |
| c1/p | = | prospective cash flow yield |
| d1/p | = | prospective dividend yield |
| b1/p | = | prospective book value yield |
| g'(e) | = | forecast growth rate of earnings per share |
| g'(c) | = | forecast growth rate of cash flow per share |
| g'(r) | = | forecast growth rate of revenue per share |
| g'(b) | = | forecast growth rate of book value per share |
| g'(e5) | = | median long-term earnings growth forecast |

The following short forms relate to a company's fundamental data (earnings per share, for example):

| | | |
|---|---|---|
| e1 | = | forecast earnings per share for the current fiscal year (the basis for the yield variable defined above). |
| e0 | = | EPS for most recent fiscal year |
| e–1 | = | EPS for the fiscal year prior to e0 |
| e–2 | = | EPS for the fiscal year prior to e–1 |
| e–3 | = | EPS for the fiscal year prior to e–2 |
| e–4 | = | EPS for the fiscal year prior to e–3 |

As needed to determine the stock's VCG score, the same notations are used for cash flow per share, revenue per share, book value per share, and dividends per share except that "c," "r," "b," or "d," respectively, are substituted for "e" in the example above.

## Appendix 4: Measuring Stock Value Orientation

*Basic Process*

A stock's value orientation reflects the price investors are willing to pay for a share of some combination of the stock's prospective earnings, dividends, sales, cash flow, and book value.

Value orientation is determined using the following three steps:

- Calculate up to five prospective yields (e1/p, d1/p, c1/p, r1/p, and b1/p) for each stock. These values are determined using the process described in the next section.
- Calculate a float market cap-weighted percentile score for each available yield factor, for each stock, within each cap band (large, mid, and small).
- Calculate a weighted average of the individual percentile scores for each stock, using the weighting scheme detailed in "Calculating Overall Value Orientation Scores" below. The weighted average score represents the strength of the stock's value orientation.

Details of each of these steps are provided below.

*Calculating Prospective Yields*

As many as possible of e1/p, d1/p, c1/p, r1/p, and b1/p are calculated for each stock. Because p is known, the method used to forecast e1, d1, and so on, is key.

If e1, c1, r1, or b1 is forecast to be negative, prospective yield on that factor is excluded for that stock. If no third-party forecast is available and e0, c0, r0, or b0 is positive, then forecast values are calculated as described below (using EPS as an example).

The relationship between prospective and current EPS is straightforward:

[1] $$e_1 = e_0 * (1 + g(e_1))$$

Because $e_0$ is known, only $g(e_1)$ must be calculated to provide a forecast of $e_1$. Also, $g(e_1)$ is calculated from historical earnings information.

First calculate as many as possible of four periodic growth rates:

[2] $$g(e)_{-4} = \left(\frac{e_0}{e_{-4}}\right)^{\frac{1}{4}} - 1$$

[3] $$g(e)_{-3} = \left(\frac{e_0}{e_{-3}}\right)^{\frac{1}{3}} - 1$$

[4] $$g(e)_{-2} = \left(\frac{e_0}{e_{-2}}\right)^{\frac{1}{2}} - 1$$

[5] $$g(e)_{-1} = \left(\frac{e_0}{e_{-1}}\right)^{1} - 1$$

Where e-1, e-2, e-3, or e-4 is negative, no growth rate is calculated using that data point. Availability for restated cash flow is limited to three years.

When as many as possible of the growth rates defined above have been calculated, average the results:

[6] $$g(e)_1 = \text{Average}\left[g(e)_{-4}, g(e)_{-3,}, g(e)_{-2}, g(e)_{-1}\right]$$

Thus:

- Estimated earnings growth g(e1) and forecast earnings (e1) are calculated only for stocks where e0 is a positive number.
- In calculating g(e1), recent growth rates are included in more of the averaged terms than are older growth rates; recent growth rates are therefore weighted more heavily than are older growth rates.
- If third-party forecasts are unavailable, e1/p, c1/p, r1/p, and b1/p are calculated in the same way.

The prospective dividend is determined based on the stock's most recent dividend and published frequency:

$$d_1 = d_0 * f_0$$

If d1/p is the only available forecast yield figure, the stock is not given a VCG assignment.

*Calculating Percentile Scores for Each Value Factor*

When one or more of e1/p, d1/p, c1/p, r1/p, and b1/p values have been calculated, with or without d1/p, each stock is assigned a float market-cap-weighted percentile score for each relevant factor. The percentile scores are calculated within the stock's cap classification.

Prospective earnings yield scores for stocks within each cap are used in the following example.

To calculate an earnings yield score for each stock in the cap band.

- Order all stocks in the cap band by their e1/p scores in ascending order.
- Determine the total float market cap of all stocks in the cap band.
- Calculate the float market-cap-weighted average of e1/p within the cap band.
- If all the stocks are within 3 weighted standard deviations of the weighted mean, no stocks are trimmed.

- If not, trim all stocks that are outside 3 weighted standard deviations outside the weighted median. View Appendix 5 for the trimming algorithm.
- Recalculate the float-weighted average e1/p for the cap band and then calculate the percentile score for each factor according to the following equation:

$$F_i = 50 * (1 + \frac{Xi - \mu}{3\sigma})$$

Where:

$F_i$ = Percentile score for individual factor
$X_i$ = Stock factor value
μ = Float market-cap-weighted factor average
$\sigma$ = Float market-cap-weighted factor standard deviation

- Trimmed stocks are then added back into the cap band and receive the score of lowest- and highest-scoring non trimmed stocks.

All of the steps in this section are then repeated for each of c1/p, r1/p, and b1/p, and d1/p.

*Calculating Overall Value Orientation Scores*

When the steps above are complete for each of the five value factors, a weighted average is calculated for each stock. In calculating the weighted average, e/p scores, if available, are assigned a weight of 50%; each of the other value factors is assigned an equal share of the remaining weight (either 50% or, if e/p is unavailable, 100%). The weighted average result is the stock's overall value orientation score.

## Appendix 5: Measuring Stock Growth Orientation

*Basic Process*

A stock's growth orientation reflects the rates at which its earnings, sales, cash flow, and book value are expected to grow. Forecast dividend growth rates are not used to determine stock growth orientation.

Determining a stock's growth orientation consists of three steps:

- For each stock, calculate as many as possible of the four average growth rates g'(e), g'(c), g'(r), and g'(b) using the process described in the next section.

- Calculate a float-weighted percentile score for each calculated growth rate, for each stock, within each cap band (large and mid), and a float-weighted percentile score for g(e5), if this is available from a third party.
- Calculate a weighted average of the individual growth rate percentile scores for each stock, using the weighting scheme detailed in the "Calculating Overall Growth Orientation Scores" section below.

The weighted average score calculated in Step 3 above represents the strength of the stock's growth orientation.

*Calculating Stock Growth Scores*

As many as possible of g'(e), g'(c), g'(r), and g'(b) are calculated for each stock. In addition, if g(e5) is available from a third party, it is used as a fifth growth rate indicator. The example growth rate calculation below uses g'(e), but the process is identical for g'(c), g'(r), and g'(b).

If e0 and e–1 are negative, then g'(e) is not calculated. If e0 or e–1 is positive, then g'(e) is calculated as follows:

First calculate as many as possible of five periodic growth rates:

[1] $$g'(e)_{-4} = \left(\frac{e_n}{e_{-4}}\right)^{\frac{1}{n+4}} - 1$$

[2] $$g'(e)_{-3} = \left(\frac{e_n}{e_{-3}}\right)^{\frac{1}{n+3}} - 1$$

[3] $$g'(e)_{-2} = \left(\frac{e_n}{e_{-2}}\right)^{\frac{1}{n+2}} - 1$$

[4] $$g'(e)_{-1} = \left(\frac{e_n}{e_{-1}}\right)^{\frac{1}{n+1}} - 1$$

[5] $$g'(e)_0 = \left(\frac{e_n}{e_0}\right)^{\frac{1}{n}} - 1$$

Where:

n = Latest period (0 or -1) in which EPS is positive

If e0, e–1, e–2, e–3, or e–4 is negative, no growth rate is calculated using that data point. If n=0, up to four rates are calculated and if n=–1, up to three growth rates are calculated.

When all available growth rates defined above have been calculated, average the results:

[6] $$g'(e) = \text{Average}\left[g'(e)_{-4}, g'(e)_{-3}, g'(e)_{-2}, g(e)_{-1}\right]$$

Revenue, cash flow, and book value growth rates are calculated in the same way.

If growth information for at least one factor, spanning at least two separate growth periods, is unavailable for a given stock, the stock is not given a VCG assignment.

*Calculating Percentile Scores for Each Growth Factor*

As with value orientation factors, the growth orientation factor scores for each stock are next translated into rescaled percentile scores. The percentile scores are calculated within the stock's cap band.

Prospective earnings growth rate scores for stocks within each cap are used in the following example.

To calculate an earnings growth score for each stock in the cap band:

- Order all stocks in the cap band by their g'(e) scores in ascending order.
- Determine the total float market cap of all stocks in the cap band.
- Calculate the float market-cap-weighted average of g'(e) within the cap band.
- If all the stocks are within 3 weighted standard deviations of the weighted mean, no stocks are trimmed.
- If not, trim all stocks that are outside 3 weighted standard deviations outside the weighted median. View Appendix 5 for the trimming algorithm.
- Recalculate the float-weighted average g'(e) for the cap band and then calculate the percentile score for each factor according to the following equation:

$$F_i = 50 * (1 + \frac{Xi - \mu}{3\sigma})$$

Where:

$F_i$ = Percentile score for individual factor
$X_i$ = Stock factor value
$\mu$ = Float market-cap-weighted factor average
$\sigma$ = Float market-cap-weighted factor standard deviation

- Trimmed stocks are then added back into the cap band and receive the score of lowest- and highest-scoring non-trimmed stocks.

All of the steps in this section are then repeated for each of the other four growth orientation factors, including g(e5).

*Calculating Overall Growth Orientation Scores*

When the above steps are completed for each of the five growth orientation factors, a weighted average is calculated for each stock. In calculating the weighted average, g(e5) scores, if available, are assigned a weight of 50%; each of the other growth factors is assigned an equal share of the remaining weight (either 50% or, if g(e5) is unavailable, 100%). The weighted average result is the stock's overall growth orientation score.

## Appendix 6: Trimming Algorithm

The following describes two processes related to calculating percentile scores for each of the 10 factors:

- Test whether trimming should be applied to datasets for individual factors.
- If so, determine what values to trim.

$w_i'$ is calculated as: $w_i' = \frac{w_i}{\sum_{j \in \Omega} w_i}$

We define the following statistics,

Weighted mean: $\mu = \sum_{i \epsilon \Omega} w_i' x_i$

Weighted standard deviation: $\sigma = \sqrt{\sum_{i \epsilon \Omega} w_i' (x_i - \mu)^2}$

Minimum and maximum: $Min[x] = Min\{x_i \mid i \in \Omega\}$
$Max[x] = Max\{x_i \mid i \in \Omega\}$

Where:

N = the number of securities
$x_i$ = the variable in the question for security i
$w_i$ = the float market cap weight of security i for the full set of N securities within the cap band
$\Omega$ = the set of securities that have not been trimmed
$\bar{\Omega}$ = the set of securities that have been trimmed
$w_i'$ = the float market cap weight of security i for $\Omega$

*Weighted Median*

To calculate the weighted median:

- Sort the values of $x_i$ $i \in \Omega$ from lowest to highest. Let M = the number of elements of $\Omega$. Let (m) denote the index for the $m^{th}$ lowest value of $x_i$ so that $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(M)}$.
- Find the smallest value of m* such that $\sum_{m=1}^{m*} w'_{(m)} \geq \frac{1}{2}$
- The median is Med Med[x] = $x_{(m*)}$

Weight in Untrimmed Set

$$p = \sum_{i \in \Omega} w_i$$

We set a threshold for p, p*=0.95

The Algorithm:

1. Set $\Omega = \{1,2,\ldots,N\}$ and $\bar{\Omega} = \Phi$.
2. Calculate $\mu$, $\sigma$, Min[x], Max[x], Med[x], and p.
3. If ($Min[x] \geq \mu - 3\sigma$ and $Max[x] \leq \mu + 3\sigma$) or $p \leq p^*$, go to step 6.

4. For each i ∈ Ω, if $x_i$ < Med[x] - 3$\sigma$ or $x_i$ > Med[x] + 3$\sigma$, move i from $\bar{\Omega}$ to Ω.
5. Go to step 2.
6. For each $i \in \bar{\Omega}$, for replace any $x_i$ < Min[x] with Min[x], and any $x_i$ > Max[x] with Max[x].

*Calculating the Score*

Using the values of $\mu$ and $\sigma$ from the final interation of the algorithm, the score of each stock is:

$$S_i = 50 * (1 + \frac{Xi - \mu}{3\sigma})$$

## Appendix 7: Assigning Stocks to a Size Segment

Stocks from the parent benchmarks are categorized into one of three market-capitalization bands - large, mid, or small. For details on this process, please refer to the Morningstar Broad Style Indexes methodology.

## Appendix 8: Determining Value and Size Tilts

Weights of each constituent in a given index are tilted toward small-/micro-cap and value, a part of the market that has historically outperformed the total U.S. market over long periods of time. The weight adjustment factors are determined as follows:

| | | |
|---|---|---|
| Large Value **LV** | Large Core **LC** | Large Growth **LG** |
| Mid Value **MV** | Mid Core **MC** | Mid Growth **MG** |
| Small Value **SV** | Small Core **SV** | Small Growth **SG** |

Let:

$W_{AB}$= the market weight of style index AB as a fraction of the parent index, A = L, M, or S; B = V., C, or G. By definition:

$$\sum_{A\in\{L,M,S,\}} \sum_{B\in\{V,C,G\}} W_{AB} = 1$$

For A = L, M, and S, is defined:

$$W_A = \sum_{B \in \{V,C,G\}} W_{AB}$$

Similarly, for B = V, C, and G, is defined:

$$W_B = \sum_{A \in \{L,M,S\}} W_{AB}$$

We also divide the market into asymmetric quadrants as follows:

$$W_I = W_{LC} + W_{LG} + W_{MC} + W_{MG}$$

$$W_{\mathrm{II}} = W_{LV} + W_{MV}$$

$$W_{\mathrm{III}} = W_{SV}$$

$$W_{\mathrm{IV}} = W_{SC} + W_{SG}$$

A parallel notation is used for the portfolio by replacing uppercase W with lowercase w. For example, $W_{MC}$ is the allocation to the Mid-Cap Core Index, $W_M$ is the mid-cap allocation, and $W_I$ is the allocation to quadrant I.

The value tilt is defined as a number between 0 and 1 that represents how much the portfolio is tilted toward value. The formula for the value tilt is:

$$\varphi_X = W_V - \frac{W_V}{W_G} W_G$$

Similarly, we define the size tilt as a number between 0 and 1 that represents how much the portfolio is tilted toward small-cap. The formula for the size tilt is:

$$\varphi_Y = W_S - \frac{W_S}{W_L} W_L$$

### Problem and Solution

Value and size tilt targets are denoted as θX and θY, respectively. To reach these targets, weight is allocated across the quadrants. It is assumed that within each quadrant, relative market weighting is maintained. Therefore:

$$W_{LV} = \frac{W_{LV}}{W_{\mathrm{II}}} W_{\mathrm{II}}$$

$$W_{LC} = \frac{W_{LC}}{W_I} W_I$$

$$W_{LG} = \frac{W_{LG}}{W_I} W_I$$

$$W_{MV} = \frac{W_{MV}}{W_{II}} W_{II}$$

$$W_{MC} = \frac{W_{MC}}{W_I} W_I$$

$$W_{MG} = \frac{W_{MG}}{W_I} W_I$$

$$W_{SV} = W_{III}$$

$$W_{SC} = \frac{W_{SC}}{W_{IV}} W_{IV}$$

$$W_{SG} = \frac{W_{SG}}{W_{IV}} W_{IV}$$

Hence, we need to determine four values; namely, $w_{I}$, $w_{II}$, $w_{III}$, and $w_{IV}$. We have three conditions to impose:

- The quadrant weights must sum to one:

$$\sum_{Q_e(\mathrm{I,II,III,}IV)} W_Q = 1$$

- The value tilt target must be met: $\varphi_X = \theta_X$
- The size tilt target must be met: $\varphi_Y = \theta_Y$

Because the three conditions are linear, three of the four quadrant weights can be expressed as a linear function of the remaining one. Express $W_{II}$ and $W_{IV}$ as functions of $W_{III}$ as follows:

$$W_{\mathrm{II}} = \alpha_{\mathrm{II}} + \beta_{\mathrm{II}} W_{\mathrm{III}}$$

$$W_{IV} = \alpha_{IV} + \beta_{IV} W_{III}$$

Where $\alpha_{II}$,$\beta_{II}$,$\alpha_{IV}$, and $\beta_{IV}$ are coefficients to be determined.

Since the quadrant weights must sum to 1:

$$W_{\mathrm{I}} = 1 - W_{\mathrm{II}} - W_{\mathrm{III}} - W_{\mathrm{IV}}$$

Thus, we can write:

$$W_{\mathrm{I}} = \alpha_{\mathrm{I}} + \beta_{\mathrm{I}} W_{\mathrm{III}}$$

Where:

$\alpha_{\mathrm{I}} = 1 - \alpha_{\mathrm{II}} - \alpha_{IV}$
$\beta_{\mathrm{I}} = -(\beta_{\mathrm{II}} + 1 + \beta_{IV})$

For completeness, is defined:

$\alpha_{\mathrm{III}}$ **=0**
$\beta_{\mathrm{III}}$**= 1**

From conditions (1), (2), and (3) the following pair of linear equations is derived in $W_{\mathrm{II}}\ and\ W_{\mathrm{IV}}$

$$a_{11}W_{\mathrm{II}} + a_{12}W_{\mathrm{IV}} = \theta_Y + \frac{W_S}{W_L}\frac{W_{LC}+W_{LG}}{W_{\mathrm{I}}} - \left(1 + \frac{W_S}{W_L}\frac{W_{LC}+W_{LG}}{W_{\mathrm{I}}}\right) W_{\mathrm{III}}$$

$$a_{21}W_{\mathrm{II}} + a_{22}W_{\mathrm{IV}} = \theta_X + \frac{W_V}{W_G}\frac{W_{MG}+W_{LG}}{W_{\mathrm{I}}} - \left(1 + \frac{W_V}{W_G}\frac{W_{MG}+W_{LG}}{W_{\mathrm{I}}}\right) W_{\mathrm{III}}$$

Where:

$$a_{11} = \frac{W_S}{W_L}\left(\frac{W_{LC}+W_{LG}}{W_I} - \frac{W_{LV}}{W_{II}}\right)$$

$$a_{12} = \frac{W_S}{W_L}\frac{W_{LC}+W_{LG}}{W_{\mathrm{I}}} + 1$$

$$a_{21} = \frac{W_V}{W_G}\frac{W_{MG}+W_{LG}}{W_{\mathrm{I}}} + 1$$

$$a_{22} = \frac{W_V}{W_G}\left(\frac{W_{MG}+W_{LG}}{W_{\mathrm{I}}} - \frac{W_{SG}}{W_{IV}}\right)$$

The solution to this pair of equations is:

$$W_{\mathrm{II}} = A_{11}\left(\theta_Y + \frac{W_S}{W_L}\frac{W_{LC}+W_{LG}}{W_{\mathrm{I}}}\right) + A_{12}\left(\theta_X + \frac{W_V}{W_G}\frac{W_{MG}+W_{LG}}{W_I}\right) - \left[A_{11}\left(1 + \frac{W_S}{W_L}\frac{W_{LC}+W_{LG}}{W_I}\right) + A_{12}\left(1 + \frac{W_V}{W_G}\frac{W_{MG}+W_{LG}}{W_I}\right)\right] W_{\mathrm{III}}$$

$$W_{IV} = A_{21}\left(\theta_Y + \frac{W_S}{W_L}\frac{W_{LC}+W_{LG}}{W_I}\right) + A_{22}\left(\theta_X + \frac{W_V}{W_G}\frac{W_{MG}+W_{LG}}{W_I}\right) - \left[A_{21}\left(1 + \frac{W_S}{W_L}\frac{W_{LC}+W_{LG}}{W_I}\right) + A_{22}\left(1 + \frac{W_V}{W_G}\frac{W_{MG}+W_{LG}}{W_I}\right)\right] W_{III}$$

Where:

$$A_{11} = \frac{a_{22}}{a_{11}a_{22} - a_{21}a_{22}}$$

$$A_{12} = \frac{-a_{12}}{a_{11}a_{22} - a_{21}a_{22}}$$

$$A_{21} = \frac{-a_{21}}{a_{11}a_{22} - a_{21}a_{22}}$$

$$A_{22} = \frac{a_{11}}{a_{11}a_{22} - a_{21}a_{22}}$$

Hence, it follows that:

$$\alpha_{\text{II}} = A_{11}\left(\theta_Y + \frac{W_S}{W_L}\frac{W_{LC} + W_{LG}}{W_I}\right) + A_{12}\left(\theta_X + \frac{W_V}{W_G}\frac{W_{MG} + W_{LG}}{W_I}\right)$$

$$\beta_{II} = -\left[A_{11}\left(1 + \frac{W_S}{W_L}\frac{W_{LC} + W_{LG}}{W_I}\right) + A_{12}\left(1 + \frac{W_V}{W_G}\frac{W_{MG} + W_{LG}}{W_I}\right)\right]$$

$$\alpha_{\text{IV}} = A_{21}\left(\theta_Y + \frac{W_S}{W_L}\frac{W_{LC} + W_{LG}}{W_I}\right) + A_{22}\left(\theta_X + \frac{W_V}{W_G}\frac{W_{MG} + W_{LG}}{W_I}\right)$$

$$\beta_{IV} = -\left[A_{21}\left(1 + \frac{W_S}{W_L}\frac{W_{LC} + W_{LG}}{W_I}\right) + A_{22}\left(1 + \frac{W_V}{W_G}\frac{W_{MG} + W_{LG}}{W_I}\right)\right]$$

We have now reduced the problem of selecting a tilted portfolio to the choice of a single variable: namely, $w_{III}$. However, the choice of $w_{III}$ is limited by the constraint that all of the quadrant weights be between 0 and 1. Since we now can express each quadrant weight as a linear function of $w_{III}$, we can find the lower and upper limits on $w_{III}$ as follows:

For each quadrant Q = I, II, III, IV, calculate:

$$L_Q = \min\left(\frac{-\alpha_Q}{\beta_Q}, \frac{1 - \alpha_Q}{\beta_Q}\right)$$

$$H_Q = \max\left(\frac{-\alpha_Q}{\beta_Q}, \frac{1 - \alpha_Q}{\beta_Q}\right)$$

Then calculate:

$$W_{\text{III}}^{min} = \underset{Q\epsilon\{\text{I,II,III,IV}\}}{max} L_Q$$

$$W_{III}^{max} = \underset{Q\epsilon\{I,II,III,IV\}}{min} H_Q$$

The first candidate for $W_{\text{III}}$ is the one that minimizes the Euclidean distance between the investor's quadrant allocations to the market quadrant allocations. The problem is:

$$\min_{W_{\text{III}}} \sum_{Q\epsilon\{I,II,III,IV\}} \left(\alpha_Q + \beta_Q w_{III} - W_Q\right)^2$$

The solution to this problem is:

$$\widehat{w}_{\text{III}} = \frac{\sum_{Q\epsilon\{I,II,III,IV\}} (W_Q - \alpha_Q)\beta_Q}{\sum_{Q\epsilon\{I,II,III,IV\}} \left(\beta_Q\right)^2}$$

If $w_{\text{III}}^{min} \leq \widehat{w}_{III} \leq w_{\text{III}}^{max}$, the problem is solved. Otherwise, there are two remaining candidates; namely $w_{III}^{min}$ and $w_{III}^{max}$. Each candidate is evaluated by the Euclidean distance between the resulting portfolio and to the market-weighted portfolio of the nonzero quadrants of the candidate solution.

Value and size tilt targets are denoted as θX and θY, respectively.

To determine the size and value target tilt factors (θX and θY) for the Morningstar Developed ex-US Factor Tilt Index, the three-factor Fama French (FF) model was considered. The three factors are: excess return on the developed market portfolio; SMB, which is the difference between the returns of a small-cap developed-market portfolio and the large-cap developed-market portfolio; and HML, which is the difference in returns between a high book/market and low book/market portfolio.

Through this statistical context, the linear regression results of monthly returns for 25 distinct developed market portfolios were considered. To construct the 25 developed portfolios, first 25 developed ex-US portfolios were constructed by incrementing the size and value tilt factors by 0.1. Recall that a size and value tilt of 0 results in no tilt—that is, a float market-cap-weighted portfolio. Next, the 25 developed ex-U.S. portfolios were combined with the Morningstar US Market Factor Tilt Index assuming a 60% allocation to U.S. and 40% allocation to developed ex-U.S. markets, rebalanced quarterly.

A size tilt factor of 0.20 and value tilt factor of 0.1 results in FF regression coefficients (the betas) over the 11-plus year history of **SMB 0.14 and HML of 0.21**. These coefficients were selected as the optimal strategy for this portfolio tilted toward small and value. At each reconstitution, the Morningstar Index Committee will review the long-term explanation of portfolio exposures within the context of the FF regression to ensure the tilt factors continue to be appropriate.

For the US Market Factor Tilt index, an equivalent process is undertaken. At each reconstitution, we select from the 25 candidate portfolios the one that has beta exposures closest to the target of **SMB=0.14 and HML=0.16.** These coefficients are selected as the optimal strategy for a U.S. market portfolio tilted toward small and value.

## Appendix 9: Individual Indexes

| Index Name | Inception Date | Performance Inception |
|---|---|---|
| Morningstar US Market Factor Tilt Index | 11-Sept-14 | 97-Jun-30 |
| Morningstar Developed Markets ex-US Factor Tilt Index | 12-Sept-12 | 01-Dec-21 |
| Morningstar Emerging Markets Factor Tilt Index | 12-Sept-12 | 01-Dec-21 |

## About Morningstar Indexes

Morningstar Indexes was built to keep up with the evolving needs of investors—and to be a leading-edge advocate for them. Our rich heritage as a transparent, investor-focused leader in data and research uniquely equips us to support individuals, institutions, wealth managers, and advisors in navigating investment opportunities across major asset classes, styles, and strategies. From traditional benchmarks and unique IP-driven indexes to index design, calculation, and distribution services, our solutions span an investment landscape as diverse as investors themselves.

## Morningstar Index Methodology Committee

The Morningstar Index Methodology Committee oversees all new index development, index methodology changes, and cessation of indexes for any indexes where Morningstar owns the intellectual property. This committee is also charged with ensuring that indexes align with Morningstar Research principles and values. The group comprises members of the index team with index research, product development, product management, client service, index implementation, and operation expertise who provide the first layer of governance over index design and methodology.

## Morningstar Indexes Operations Committee

The Morningstar Index Operations Committee governs the processes, systems, and exception handling of the day-to-day management of all live indexes, including index rebalancing and reconstitution, restatements, market classification, and contingency management. The committee oversees the annual review of index methodology (as required by U.K. and EU benchmark regulations, or BMR), ensuring that methodologies remain fit for purpose and continue to achieve their stated investment objectives. The group comprises members of the index team with data, operations, corporate actions, product development, index launch, client service, and index management experience who provide the first layer of governance over index operations.

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